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稠油热采三重介质三渗模型及压力动态分析

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摘　要:为了研究碳酸盐岩储层稠油热采的井底压力变化特征,考虑温度对油度的影响和地层的应力敏感性,建立了三重介质三渗数学模型。数学模型为非线性变系数偏微分方程组,采用全隐式有限差分格式离散,用 Newton-Raphson 迭代法求解非线性差分方程组。差分方程组的 Jacobi 矩阵为三对角矩阵。利用三对角矩阵的性质求逆阵提高了迭代速度。根据计算结果绘制了试井样板曲线,进行了参数敏感性分析。研究结果表明:压力响应分为 6 个阶段;不同储层中稠油的黏度依赖于温度变化的敏感性不同。具体表现为黏度对温度变化的敏感性越高则储层中压力的数值越大,各窜流阶段出现得越早,而且会缩短溶洞向裂缝窜流进行的时间并减弱其效果;造成压力导数提前衰减。

关键词:三重介质;碳酸盐岩;稠油;变系数方程组;试井

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Triple permeability model in triple media and pressure dynamic analysis of thermal recovery of heavy oil

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Abstract: To study the characteristics of the wellbore pressure changes of thermal recovery of heavy oil in carbonate reservoirs, considering the effect of temperature on viscosity of oil and stress sensitivity of reservoirs, the triple permeability mathematical model in triple media has been constructed. The mathematical model is of nonlinear partial differential equations with variable coefficients. The difference equations have been discretized by the fully implicit finite difference scheme. Newton-Raphson method has been used for solving nonlinear difference equations. The Jacobi matrices of the difference equations are tridiagonal matrix. Using the properties of tridiagonal matrix, the iteration speed is improved. According to the calculation results, the typical well test curves have been plotted. Sensitivity of parameters has been analyzed. There are eight stages of pressure response. The sensitivities of viscosity depend on temperature are different in different reservoirs. The higher the sensitivities of viscosity depend on temperature, the greater the values of pressure in reservoirs. When the cross flow stages are earlier, the time of the cross flow between the frac-

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tures and vugs is shortened and the strength is weakened, causing the earlier decrease in the pressure derivative.

Key words: triple media; carbonate; heavy oil; equation with variable coefficient; well testing

一般碳酸盐岩储层有裂缝和溶洞发育,其渗流规律不同于只含有一种孔隙度的油气藏。碳酸盐岩渗流流动规律比单一孔隙储层复杂,因为基质系统(以下简称基质)、裂缝系统(以下简称裂缝)和溶洞系统(以下简称溶洞)间存在流体交换。其流体流动规律比单一孔隙储层复杂。为此,为真实反映碳酸盐油气藏的储层特征,需将碳酸盐岩简化为多重介质进行研究。文献[1-2]提出了双重介质模型。文献[3]对三重介质模型的建立做了基础性工作。文献[4]提出了三重孔隙介质概念。文献[5]指出三重介质曲线拟合必须考虑溶洞的影响。文献[6-9]对三渗模型进行了研究,并考虑了地层应力敏感性和二次压力梯度的影响。文献[10-11]建立并求解了考虑启动压力梯度的分形三重介质油藏数学模型。文献[12]研究了三重介质地层中油水两相非稳态窜流的水平井模型。文献[13]考虑压敏和地层渗透率各向异性,建立并求解了斜井数学模型。文献[14]建立并求解了井筒与大溶洞连通,并且考虑了流动和波动耦合的模型。

随着石油工业的发展,发现了许多碳酸盐岩类稠油油藏,稠油热采被广泛应用。文献[15]首先对碳酸盐岩稠油热采的多重介质模型进行研究,把储层分为蒸汽波及区和冷油区,建立并求解了复合油藏水平井试井解释模型。但是注蒸汽改造后,温度对流体黏度的影响从井筒到油藏边界是连续变化的,而且基质、裂缝和溶洞的渗透性需要被考虑。

目前,考虑到以上因素的多重介质试井模型还没有被提出。本研究考虑温度对流体黏度的影响,并且考虑地层应力敏感性和二次梯度项的影响,建立三孔三渗模型,用数值方法求解并对影响井底压力的因素进行分析。

1　数学模型

碳酸盐岩地层中裂缝和溶洞发育,流体通过基质、裂缝和溶洞流入井筒。3 种流体系统间存在流体交换。在稠油热采过程中,蒸汽的注入使近井地带和远井地带的温度产生差异。温度的改变使流体流动性发生变化。

假设温度的变化仅对稠油的黏度产生影响,考虑单层圆形油藏中心一口井定量生产,因内外边界温度差异而引起储层内的热传导。忽略因流体流动产生的热对流;每种介质的孔隙度随压力的变化是相对独立的;基质、裂缝和溶洞被流体所饱和;流体在 3 种介质中的流动都满足 Darcy 定律。开井前地层中各处压力等于原始地层压力 p_0;裂缝和溶洞发育良好,裂缝的渗透率大于溶洞的渗透率;基质中的流体向裂缝和溶洞发生窜流,溶洞中的流体向裂缝发生窜流,窜流是拟稳态的。基于这些假设,本研究考虑压力敏感效应和二次梯度项。数学推导和公式中的物理量符号中,下标 1 代表裂缝,下标 2 代表溶洞,下标 3 代表基质。三重介质三渗模型简图如图 1 所示。

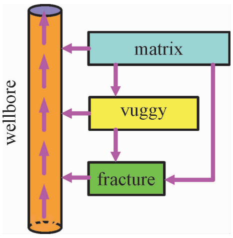

图 1　三重介质三渗模型简图

Fig. 1　Triple permeability flow scheme in triple-porosity media

假设注入蒸汽时温度不变,外边界考虑的是储层温度,看作温度是恒定的,内边界假设注热井的温度是恒定的。设井筒半径为 r_w,储层半径为 R;井筒处的温度为 T_w,储层外边界的温度为 T_R。根据稳态热传导方程,储层内的温度分布满足下列定解问题及其边界条件。

$$\begin{cases} \dfrac{d}{dr}\left(r\dfrac{dT}{dr}\right)=0 \\ r=r_w,T=T_w \\ r=R,T=T_R \end{cases} \tag{1}$$

其中:r 为半径,单位为 m;T 为温度,单位为℃。

解式(1),得

$$T=T_w+\frac{T_R-T_w}{\ln(R/r_w)}\ln(r/r_w) \tag{2}$$

定义黏度变化因子为

$$\beta=\frac{1}{\mu}\frac{d\mu}{dT} \tag{3}$$

其中：μ 为流体黏度，单位为 Pa·s；β 的单位为℃$^{-1}$。黏度变化因子表示流体黏度依赖于温度变化的敏感性。

由式(3)得

$$\mu=\mu_0\exp[\beta(T-T_R)] \tag{4}$$

其中 μ_0 为初始温度时的黏度，单位为 Pa·s。取 β 的值为 $-0.031\,584\,5$ ℃$^{-1}$。

将式(2)代入式(4)，得

$$\mu=\mu_0\exp\left[\beta\left(T_{\mathrm{w}}-T_R+\frac{T_R-T_{\mathrm{w}}}{\ln(R/r_{\mathrm{w}})}\ln(r/r_{\mathrm{w}})\right)\right] \tag{5}$$

考虑压力敏感效应，渗透率模数定义为[16]

$$\gamma_i=\frac{1}{K_i}\frac{\mathrm{d}K_i}{\mathrm{d}p_i}, i=1,2,3 \tag{6}$$

其中 $K_i(i=1,2,3)$ 为渗透率，单位为 m^2。假设 γ_i 为常数，单位为 Pa^{-1}。则

$$K_i=K_{i0}\exp[\gamma_i(p_i-p_0)], i=1,2,3 \tag{7}$$

其中 $K_{i0}(i=1,2,3)$ 为初始渗透率。$p_i(i=1,2,3)$ 为 3 种介质中的流体压力。

窜流方程为

$$\begin{cases} q_{12}=\dfrac{\alpha_{12}\rho_0}{\mu}(p_2-p_1) \\ q_{13}=\dfrac{\alpha_{13}\rho_0}{\mu}(p_3-p_1) \\ q_{23}=\dfrac{\alpha_{23}\rho_0}{\mu}(p_3-p_2) \end{cases} \tag{8}$$

其中：α_{12} 为裂缝与溶洞之间的量纲为一的窜流系数；α_{13} 为基质与裂缝之间的量纲为一的窜流系数；α_{23} 为溶洞与基质之间的量纲为一的窜流系数；q_{12}、q_{13}、q_{23} 分别为对应的窜流量，单位为 kg/(m^3·s)。

将运动方程、窜流方程和状态方程代入连续性方程(具体推导过程从略)，得到描述考虑温度影响的三重介质三渗渗流的偏微分方程组，即

$$\begin{cases} K_{10}\left[\dfrac{\partial^2 p_1}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial p_1}{\partial r}+\gamma_1\left(\dfrac{\partial p_1}{\partial r}\right)^2\right]+\alpha_{12}\mathrm{e}^{\gamma_1(p_0-p_1)}(p_2-p_1)+\alpha_{13}\mathrm{e}^{\gamma_1(p_0-p_1)}(p_3-p_1) \\ \qquad =\mu_0\varphi_1 c_{t1}\mathrm{e}^{\beta\left[T_{\mathrm{w}}-T_R+\frac{T_R-T_{\mathrm{w}}}{\ln(R/r_{\mathrm{w}})}\ln(r/r_{\mathrm{w}})\right]+\gamma_1(p_0-p_1)}\dfrac{\partial p_1}{\partial t} \\ K_{20}\left[\dfrac{\partial^2 p_2}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial p_2}{\partial r}+\gamma_2\left(\dfrac{\partial p_2}{\partial r}\right)^2\right]+\alpha_{12}\mathrm{e}^{\gamma_2(p_0-p_2)}(p_1-p_2)+\alpha_{23}\mathrm{e}^{\gamma_2(p_0-p_2)}(p_3-p_2) \\ \qquad =\mu_0\varphi_2 c_{t2}\mathrm{e}^{\beta\left[T_{\mathrm{w}}-T_R+\frac{T_R-T_{\mathrm{w}}}{\ln(R/r_{\mathrm{w}})}\ln(r/r_{\mathrm{w}})\right]+\gamma_2(p_0-p_2)}\dfrac{\partial p_2}{\partial t} \\ K_{30}\left[\dfrac{\partial^2 p_3}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial p_3}{\partial r}+\gamma_3\left(\dfrac{\partial p_3}{\partial r}\right)^2\right]+\alpha_{13}\mathrm{e}^{\gamma_3(p_0-p_3)}(p_1-p_3)+\alpha_{23}\mathrm{e}^{\gamma_3(p_0-p_3)}(p_2-p_3) \\ \qquad =\mu_0\varphi_3 c_{t3}\mathrm{e}^{\beta\left[T_{\mathrm{w}}-T_R+\frac{T_R-T_{\mathrm{w}}}{\ln(R/r_{\mathrm{w}})}\ln(r/r_{\mathrm{w}})\right]+\gamma_3(p_0-p_3)}\dfrac{\partial p_3}{\partial t} \end{cases} \tag{9}$$

其中：$\varphi_i(i=1,2,3)$ 为孔隙度；$c_{ti}(i=1,2,3)$ 为综合压缩系数，单位为 Pa^{-1}；t 为时间，单位为 s。该模型综合考虑了基质、裂缝和溶洞的渗透性，并且考虑了地层的应力敏感性和二次梯度项的影响。考虑温度对流体黏度的影响，这种影响从井底到外边界连续变化，没有把储层分区。

初始条件为

$$p_i(r,t)|_{t=0}=p_0(r_{\mathrm{w}}\leqslant r\leqslant R; i=1,2,3) \tag{10}$$

外边界条件为定压边界条件，即

$$p_i(r,t)|_{r=R}=p_0(t>0; i=1,2,3) \tag{11}$$

考虑井筒储集和表皮因子的内边界条件为

$$\frac{2\pi r_{\mathrm{w}}h}{\mu}\left(K_1\mathrm{e}^{\gamma_1(p_1-p_0)}\frac{\partial p_1}{\partial r}+K_2\mathrm{e}^{\gamma_2(p_2-p_0)}\frac{\partial p_2}{\partial r}+K_3\mathrm{e}^{\gamma_3(p_3-p_0)}\frac{\partial p_3}{\partial r}\right)\bigg|_{r=r_{\mathrm{w}}}-C\frac{dp_{\mathrm{w}}}{dt}=q, (t>0) \tag{12}$$

$$p_{\mathrm{w}}=p_1-S\frac{q\mu}{2\pi K_1 h} \tag{13}$$

$$p_{\mathrm{w}}=p_2-S\frac{q\mu}{2\pi K_2 h} \tag{14}$$

$$p_{\mathrm{w}}=p_3-S\frac{q\mu}{2\pi K_3 h} \tag{15}$$

其中：p_{w} 为井底压力，单位为 Pa；h 为储层厚度，单位为 m；q 为流量，单位为 m^3/s；C 为井筒储集系数，单位为 m^4·s^2/kg；S 为表皮系数。

按表 1 定义无因次变量，式(9)化为

$$D_1\left[\frac{\partial^2 p_{\mathrm{D1}}}{\partial r_{\mathrm{D}}^2}+\frac{1}{r_{\mathrm{D}}}\frac{\partial p_{\mathrm{D1}}}{\partial r_{\mathrm{D}}}-\gamma_{\mathrm{D1}}\left(\frac{\partial p_{\mathrm{D1}}}{\partial r_{\mathrm{D}}}\right)^2\right]+\lambda_{12}\mathrm{e}^{\gamma_{\mathrm{D1}}p_{\mathrm{D1}}}(p_{\mathrm{D2}}-p_{\mathrm{D1}})+\lambda_{13}\mathrm{e}^{\gamma_{\mathrm{D1}}p_{\mathrm{D1}}}(p_{\mathrm{D3}}-p_{\mathrm{D1}})$$

$$=\omega_1\mathrm{e}^{\beta_{\mathrm{D}}\left[\theta-\frac{\theta}{\ln(R/r_{\mathrm{w}})}\ln r_{\mathrm{D}}\right]+\gamma_{\mathrm{D1}}p_{\mathrm{D1}}}\frac{\partial p_{\mathrm{D1}}}{\partial t_{\mathrm{D}}},$$

$$D_2\left[\frac{\partial^2 p_{D2}}{\partial r_D^2}+\frac{1}{r_D}\frac{\partial p_{D2}}{\partial r_D}-\gamma_{D2}\left(\frac{\partial p_{D2}}{\partial r_D}\right)^2\right]+$$
$$\lambda_{12}e^{\gamma_{D2}p_{D2}}(p_{D1}-p_{D2})+\lambda_{23}e^{\gamma_{D2}p_{D2}}(p_{D3}-p_{D2})$$
$$=\omega_2 e^{\beta_D\left[\theta-\frac{\theta}{\ln(R/r_w)}\ln r_D\right]+\gamma_{D2}p_{D2}}\frac{\partial p_{D2}}{\partial t_D},$$
$$D_3\left[\frac{\partial^2 p_{D3}}{\partial r_D^2}+\frac{1}{r_D}\frac{\partial p_{D3}}{\partial r_D}-\gamma_{D3}\left(\frac{\partial p_{D3}}{\partial r_D}\right)^2\right]+$$
$$\lambda_{13}e^{\gamma_{D3}p_{D3}}(p_{D1}-p_{D3})+\lambda_{23}e^{\gamma_{D3}p_{D3}}(p_{D2}-p_{D3})$$
$$=\omega_3 e^{\beta_D\left[\theta-\frac{\theta}{\ln(R/r_w)}\ln r_D\right]+\gamma_{D3}p_{D3}}\frac{\partial p_{D3}}{\partial t_D}\tag{16}$$

式(16)是非线性变系数偏微分方程组，需要结合初始条件与边界条件进行数值求解。

表 1　无因次变量定义表

Tab. 1　Definitions of dimensionless variables

变量名	符号 ($i=1,2,3$)	表达式 ($i=1,2,3$)
压力	p_{Di}	$\dfrac{2\pi(k_{10}+k_{20}+k_{30})h}{\mu_0 q}(p_0-p_i)$
井底压力	p_{wD}	$\dfrac{2\pi(k_{10}+k_{20}+k_{30})h}{\mu_0 q}(p_0-p_w)$
时间	t_D	$\dfrac{(k_{10}+k_{20}+k_{30})t}{\mu r_w^2(\varphi_1 c_{t1}+\varphi_2 c_{t2}+\varphi_3 c_{t3})}$
径向距离	r_D	$\dfrac{r}{r_w}$
渗透率模数[7]	γ_{Di}	$\dfrac{\mu q\gamma_i}{2\pi h(k_{10}+k_{20}+k_{30})}$
孔洞窜流系数	λ_{12}	$\dfrac{\alpha_{12}r_w^2}{k_{10}+k_{20}+k_{30}}$
孔缝窜流系数	λ_{13}	$\dfrac{\alpha_{13}r_w^2}{k_{10}+k_{20}+k_{30}}$
洞缝窜流系数	λ_{23}	$\dfrac{\alpha_{23}r_w^2}{k_{10}+k_{20}+k_{30}}$
储容比	ω_i	$\dfrac{\varphi_i c_{ti}}{\varphi_1 c_{t1}+\varphi_2 c_{t2}+\varphi_3 c_{t3}}$
井筒储集系数	C_D	$\dfrac{C}{2\pi h r_w^2(\varphi_1 c_{t1}+\varphi_2 c_{t2}+\varphi_3 c_{t3})}$
渗透率比	D_i	$\dfrac{k_{i0}}{k_{10}+k_{20}+k_{30}}$
黏度变化因子	β_D	βT_R
无因次温度	θ	$\dfrac{T_w-T_R}{T_R}$

初始条件为

$$p_{Di}(r_D,t_D)\big|_{t_D=0}=0,\ (1\leqslant r_D\leqslant R/r_w;i=1,2,3)\tag{17}$$

外边界条件为

$$p_{Di}(r_D,t_D)\big|_{r_D=R/r_w}=0,\ (t_D>0;i=1,2,3)\tag{18}$$

内边界条件为

$$C_D\frac{dp_{wD}}{dt_D}-\left(D_1e^{-\gamma_{D1}p_{D1}}\frac{\partial p_{D1}}{\partial r_D}+D_2e^{-\gamma_{D2}p_{D2}}\frac{\partial p_{D2}}{\partial r_D}+\right.$$
$$\left.D_3e^{-\gamma_{D3}p_{D3}}\frac{\partial p_{D3}}{\partial r_D}\right)\Bigg|_{r_D=1}=1,\ (t_D>0)\tag{19}$$

$$p_{wD}=\left(p_{D1}-r_DSe^{-\gamma_{D1}p_{D1}}\frac{\partial p_{D1}}{\partial r_D}\right)\Bigg|_{r_D=1}\tag{20}$$

$$p_{wD}=\left(p_{D2}-r_DSe^{-\gamma_{D2}p_{D2}}\frac{\partial p_{D2}}{\partial r_D}\right)\Bigg|_{r_D=1}\tag{21}$$

$$p_{wD}=\left(p_{D3}-r_DSe^{-\gamma_{D3}p_{D3}}\frac{\partial p_{D3}}{\partial r_D}\right)\Bigg|_{r_D=1}\tag{22}$$

为了消去二次梯度项，对方程组及其初始条件和边界条件进行 Pedrosa 代换[16]，则

$$p_{D1}(r_D,t_D)=-\frac{1}{\gamma_{D1}}\ln[1-\gamma_{D1}\eta(r_D,t_D)]\tag{23}$$

$$p_{D2}(r_D,t_D)=-\frac{1}{\gamma_{D2}}\ln[1-\gamma_{D2}\xi(r_D,t_D)]\tag{24}$$

$$p_{D3}(r_D,t_D)=-\frac{1}{\gamma_{D3}}\ln[1-\gamma_{D3}\zeta(r_D,t_D)]\tag{25}$$

$$x=\ln r_D\tag{26}$$

式中 η、ξ 和 ζ 表示变换后的压力。

经 Pedrosa 代换后消去了二次梯度项，方程组变为如下形式。

$$\frac{D_1}{e^{2x}}\frac{\partial^2\eta}{\partial x^2}=\omega_1(1-\gamma_{D1}\eta)^{-1}e^{\beta_D\left[\theta-\frac{\theta}{\ln(R/r_w)}x\right]}\frac{\partial\eta}{\partial t_D}+$$
$$\lambda_{12}\left[\frac{\ln(1-\gamma_{D2}\xi)}{\gamma_{D2}}-\frac{\ln(1-\gamma_{D1}\eta)}{\gamma_{D1}}\right]+$$
$$\lambda_{13}\left[\frac{\ln(1-\gamma_{D3}\zeta)}{\gamma_{D3}}-\frac{\ln(1-\gamma_{D1}\eta)}{\gamma_{D1}}\right],$$
$$\frac{D_2}{e^{2x}}\frac{\partial^2\xi}{\partial x^2}=\omega_2(1-\gamma_{D2}\xi)^{-1}e^{\beta_D\left[\theta-\frac{\theta}{\ln(R/r_w)}x\right]}\frac{\partial\xi}{\partial t_D}+$$
$$\lambda_{12}\left[\frac{\ln(1-\gamma_{D1}\eta)}{\gamma_{D1}}-\frac{\ln(1-\gamma_{D2}\xi)}{\gamma_{D2}}\right]+$$
$$\lambda_{23}\left[\frac{\ln(1-\gamma_{D3}\zeta)}{\gamma_{D3}}-\frac{\ln(1-\gamma_{D2}\xi)}{\gamma_{D2}}\right],$$
$$\frac{D_3}{e^{2x}}\frac{\partial^2\zeta}{\partial x^2}=\omega_3(1-\gamma_{D3}\zeta)^{-1}e^{\beta_D\left[\theta-\frac{\theta}{\ln(R/r_w)}x\right]}\frac{\partial\zeta}{\partial t_D}+$$
$$\lambda_{13}\left[\frac{\ln(1-\gamma_{D1}\eta)}{\gamma_{D1}}-\frac{\ln(1-\gamma_{D3}\zeta)}{\gamma_{D3}}\right]+$$
$$\lambda_{23}\left[\frac{\ln(1-\gamma_{D2}\xi)}{\gamma_{D2}}-\frac{\ln(1-\gamma_{D3}\zeta)}{\gamma_{D3}}\right]\tag{27}$$

初始条件为

$$\eta\big|_{t_D=0}=\xi\big|_{t_D=0}=\zeta\big|_{t_D=0}=0 \tag{28}$$

外边界条件为

$$\eta\big|_{x=\ln(R/r_w)}=\xi\big|_{x=\ln(R/r_w)}=\zeta\big|_{x=\ln(R/r_w)}=0 \tag{29}$$

内边界条件为

$$C_D\frac{\mathrm{d}p_{wD}}{\mathrm{d}t_D}-\left(D_1\frac{\partial\eta}{\partial x}+D_2\frac{\partial\xi}{\partial x}+D_3\frac{\partial\zeta}{\partial x}\right)\Bigg|_{x=0}=1 \tag{30}$$

$$p_{wD}=\left[-\frac{\ln(1-\gamma_{D1}\eta)}{\gamma_{D1}}-S\frac{\partial\eta}{\partial x}\right]\Bigg|_{x=0} \tag{31}$$

$$p_{wD}=\left[-\frac{\ln(1-\gamma_{D2}\xi)}{\gamma_{D2}}-S\frac{\partial\xi}{\partial x}\right]\Bigg|_{x=0} \tag{32}$$

$$p_{wD}=\left[-\frac{\ln(1-\gamma_{D3}\zeta)}{\gamma_{D3}}-S\frac{\partial\zeta}{\partial x}\right]\Bigg|_{x=0} \tag{33}$$

2 数值解法

一阶导数采用向前差商，二阶导数采用中心差商，对变系数方程组进行全隐式差分离散[17-18]，得

$$\begin{aligned}
&\frac{D_1}{e^{2(j-1)\Delta x}}\frac{(\eta_{j-1}^{n+1}-2\eta_j^{n+1}+\eta_{j+1}^{n+1})}{(\Delta x)^2}\\
&=\frac{\omega_1}{(1-\gamma_{D1}\eta_j^{n+1})}e^{\beta_D\left[\theta-\frac{\theta}{\ln(R/r_w)}(j-1)\Delta x\right]}\frac{\eta_j^{n+1}-\eta_j^n}{\Delta t}+\\
&\lambda_{12}\left[\frac{\ln(1-\gamma_{D2}\xi_j^n)}{\gamma_{D2}}-\frac{\ln(1-\gamma_{D1}\eta_j^{n+1})}{\gamma_{D1}}\right]+\\
&\lambda_{13}\left[\frac{\ln(1-\gamma_{D3}\zeta_j^n)}{\gamma_{D3}}-\frac{\ln(1-\gamma_{D1}\eta_j^{n+1})}{\gamma_{D1}}\right],\\
&\frac{D_2}{e^{2(j-1)\Delta x}}\frac{(\xi_{j-1}^{n+1}-2\xi_j^{n+1}+\xi_{j+1}^{n+1})}{(\Delta x)^2}\\
&=\frac{\omega_2}{(1-\gamma_{D2}\xi_j^{n+1})}e^{\beta_D\left[\theta-\frac{\theta}{\ln(R/r_w)}(j-1)\Delta x\right]}\frac{\xi_j^{n+1}-\xi_j^n}{\Delta t}+\\
&\lambda_{12}\left[\frac{\ln(1-\gamma_{D1}\eta_j^n)}{\gamma_{D1}}-\frac{\ln(1-\gamma_{D2}\xi_j^{n+1})}{\gamma_{D2}}\right]+\\
&\lambda_{23}\left[\frac{\ln(1-\gamma_{D3}\zeta_j^n)}{\gamma_{D3}}-\frac{\ln(1-\gamma_{D2}\xi_j^{n+1})}{\gamma_{D2}}\right],\\
&\frac{D_3}{e^{2(j-1)\Delta x}}\frac{(\zeta_{j-1}^{n+1}-2\zeta_j^{n+1}+\zeta_{j+1}^{n+1})}{(\Delta x)^2}\\
&=\frac{\omega_3}{(1-\gamma_{D3}\zeta_j^{n+1})}e^{\beta_D\left[\theta-\frac{\theta}{\ln(R/r_w)}(j-1)\Delta x\right]}\frac{\zeta_j^{n+1}-\zeta_j^n}{\Delta t}+\\
&\lambda_{23}\left[\frac{\ln(1-\gamma_{D2}\xi_j^n)}{\gamma_{D2}}-\frac{\ln(1-\gamma_{D3}\zeta_j^{n+1})}{\gamma_{D3}}\right]+\\
&\lambda_{13}\left[\frac{\ln(1-\gamma_{D1}\eta_j^n)}{\gamma_{D1}}-\frac{\ln(1-\gamma_{D3}\zeta_j^{n+1})}{\gamma_{D3}}\right]
\end{aligned}\tag{34}$$

其中：n 从 0 取到 $T-1$；j 从 1 取到 $J-1$；J 代表步长的最大值。

初始条件离散为

$$\eta_j^0=\xi_j^0=\zeta_j^0=0 \tag{35}$$

外边界条件离散为

$$\eta_J^n=\xi_J^n=\zeta_J^n=0 \tag{36}$$

内边界条件经过文献[7]中的处理离散为

$$\begin{aligned}
&\frac{C_D}{\Delta t}\left[-\frac{\ln(1-\gamma_{D1}\eta_1^{n+1})}{\gamma_{D1}}-S\frac{\eta_1^{n+1}-\eta_0^{n+1}}{\Delta x}\right]-\\
&D_1\frac{\eta_1^{n+1}-\eta_0^{n+1}}{\Delta x}-D_2\frac{\xi_1^n-\xi_0^n}{\Delta x}-D_3\frac{\zeta_1^n-\zeta_0^n}{\Delta x}+\\
&\frac{C_D}{\Delta t}\left[\frac{\ln(1-\gamma_{D1}\eta_1^n)}{\gamma_{D1}}+S\frac{\eta_1^n-\eta_0^n}{\Delta x}\right]-1=0
\end{aligned}\tag{37}$$

$$\begin{aligned}
&\frac{C_D}{\Delta t}\left[-\frac{\ln(1-\gamma_{D2}\xi_1^{n+1})}{\gamma_{D2}}-S\frac{\xi_1^{n+1}-\xi_0^{n+1}}{\Delta x}\right]-\\
&D_1\frac{\eta_1^n-\eta_0^n}{\Delta x}-D_2\frac{\xi_1^{n+1}-\xi_0^{n+1}}{\Delta x}-D_3\frac{\zeta_1^n-\zeta_0^n}{\Delta x}+\\
&\frac{C_D}{\Delta t}\left[\frac{\ln(1-\gamma_{D2}\xi_1^n)}{\gamma_{D2}}+S\frac{\xi_1^n-\xi_0^n}{\Delta x}\right]-1=0
\end{aligned}\tag{38}$$

$$\begin{aligned}
&\frac{C_D}{\Delta t}\left[-\frac{\ln(1-\gamma_{D3}\zeta_1^{n+1})}{\gamma_{D3}}-S\frac{\zeta_1^{n+1}-\zeta_0^{n+1}}{\Delta x}\right]-\\
&D_1\frac{\eta_1^n-\eta_0^n}{\Delta x}-D_2\frac{\xi_1^n-\xi_0^n}{\Delta x}-D_3\frac{\zeta_1^{n+1}-\zeta_0^{n+1}}{\Delta x}+\\
&\frac{C_D}{\Delta t}\left[\frac{\ln(1-\gamma_{D3}\zeta_1^n)}{\gamma_{D3}}+S\frac{\zeta_1^n-\zeta_0^n}{\Delta x}\right]-1=0
\end{aligned}\tag{39}$$

式(34)中的 3 个非线性变系数差分方程与其相应的边界条件式(37)、式(38)和式(39)组成 3 个方程组。采用 Newton-Raphson 迭代法进行求解。

以式(34)中第一式与式(37)组成的方程组为例。设方程具有如下形式。

$$\begin{cases}f_1(\eta_0^{n+1},\eta_1^{n+1},\cdots,\eta_{J-1}^{n+1})=0\\ \cdots\\ f_J(\eta_0^{n+1},\eta_1^{n+1},\cdots,\eta_{J-1}^{n+1})=0\end{cases}\tag{40}$$

这里 f_1 为式(37)的左端，$f_i(i=2\sim J)$ 由式(34)中第 1 式移项得到。用向量记号记为

$$\boldsymbol{\eta}^{n+1}=(\eta_0^{n+1},\eta_1^{n+1},\cdots,\eta_{J-1}^{n+1})^{\mathrm{T}}\in\boldsymbol{R}^J \tag{41}$$

$$\boldsymbol{F}=(f_1,\cdots,f_J)^{\mathrm{T}} \tag{42}$$

则方程组可以写为

$$\boldsymbol{F}(\boldsymbol{\eta}^{n+1})=0 \tag{43}$$

Jacobi 矩阵为

$$\boldsymbol{J}(\boldsymbol{\eta}^{n+1})=\begin{bmatrix}J_{11}&\cdots&J_{1J}\\ \vdots&\ddots&\vdots\\ J_{J1}&\cdots&J_{JJ}\end{bmatrix}\tag{44}$$

其中

$$J_{11}=\frac{\partial f_1}{\partial \eta_0^{n+1}}=\frac{\left(\frac{C_{\mathrm{D}}S}{\Delta t}+D_1\right)}{\Delta x},$$

$$J_{12}=\frac{\partial f_1}{\partial \eta_1^{n+1}}=-\frac{\left(\frac{C_{\mathrm{D}}S}{\Delta t}+D_1\right)}{\Delta x}+\frac{C_{\mathrm{D}}}{\Delta t}\cdot\frac{1}{1-\gamma_{\mathrm{D1}}\eta_1^{n+1}},$$

$$J_{i,i-1}=\frac{\partial f_i}{\partial \eta_{j-1}^{n+1}}=J_{i,i+1}=\frac{\partial f_i}{\partial \eta_{j+1}^{n+1}}=\frac{D_1}{\mathrm{e}^{2(j-1)\Delta x}(\Delta x)^2},$$

$$(i=2,3,\cdots,J-1;j=i-1),$$

$$J_{J,J-1}=\frac{\partial f_J}{\partial \eta_{J-2}^{n+1}}=\frac{D_1}{\mathrm{e}^{2(J-2)\Delta x}(\Delta x)^2},$$

$$J_{ii}=-2J_{i,i-1}-\frac{\omega_1}{\Delta t(1-\gamma_{\mathrm{D1}}\eta_j^{n+1})}\cdot \mathrm{e}^{\beta_{\mathrm{D}}\left[\theta-\frac{\theta(j-1)\Delta x}{\ln(R/r_{\mathrm{w}})}\right]}\cdot$$

$$[\gamma_{\mathrm{D1}}(\eta_j^{n+1}-\eta_j^{n})+1]-(\lambda_{12}+\lambda_{13})/(1-\gamma_{\mathrm{D1}}\eta_j^{n+1}),$$

$$(i=2,3,\cdots,J;j=i-1)$$

经过迭代后的新值为

$$(\boldsymbol{\eta}^{n+1})^{(k+1)}=(\boldsymbol{\eta}^{n+1})^{(k)}-(\boldsymbol{J}(\boldsymbol{\eta}^{n+1})^{-1})^{(k)}(\boldsymbol{F}(\boldsymbol{\eta}^{n+1}))^{(k)} \quad (45)$$

其中 k 为迭代次数。

计算过程中每一次迭代都要计算 Jacobi 矩阵的逆矩阵。由于 Jacobi 矩阵为三对角矩阵,利用三对角矩阵的特点可以使求逆矩阵更简单。采用文献[19]中对三对角矩阵求逆的算法提高了计算速度。

3 结果分析

描述温度影响下的三重介质三渗渗流的偏微分方程组是变系数非线性方程组,离散以后的计算迭代次数比不考虑温度影响的方程组迭代次数多,这种现象随着 β_{D}绝对值的增大而越来越明显。

在双对数坐标中画出井底压力及压力导数曲线,如图 2 所示。可以划分为 6 个流动阶段:阶段 1 为井筒储存阶段,压力和压力导数曲线呈单位斜率直线;阶段 2 为表皮因子影响阶段,压力导数曲线呈驼峰状;阶段 3 为溶洞向裂缝窜流阶段,压力导数曲线出现凹陷;阶段 4 为基质向裂缝窜流阶段,压力导数曲线出现凹陷;阶段 5 为基质向溶洞窜流阶段,压力导数曲线出现凹陷;阶段 6 为稳定流动阶段,压力导数急剧下降,井底压力图线为水平直线。

对计算结果进行参数敏感性分析如下。

1)由于不同储层中稠油对温度变化的敏感性不同,β_{D}绝对值也有所不同。注蒸汽对储层进行改造后,注蒸汽后流体的流动性增强,定量生产时消耗的地层能量低,井底压力升高,各窜流阶段出现得越早,而且会缩短溶洞向裂缝窜流进行的时间并减弱其效果;压力导数提前衰减。这些影响会随着稠油对温度的敏感性的增强而越来越明显,如图 3 所示。

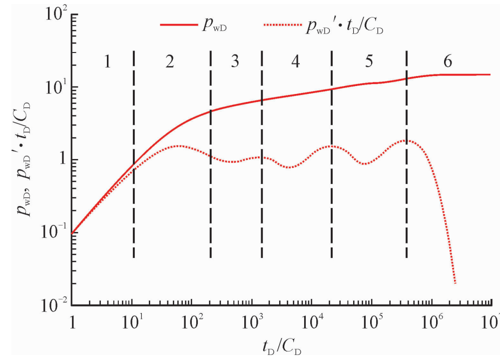

图 2 井底压力及压力导数曲线

Fig. 2 The wellbore pressure and pressure-derivative curves

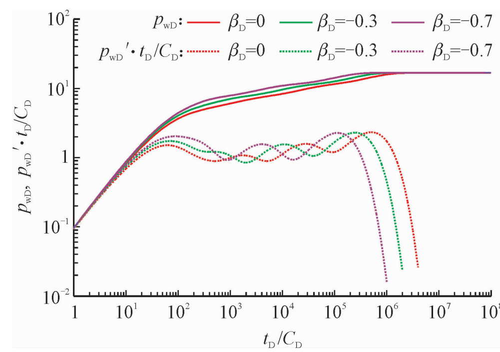

图 3 β_{D}对无因次井底压力及压力导数的影响

Fig. 3 The effect of β_{D} on dimensionless wellbore pressure and pressure-derivative

2)渗透率模数越大,渗透率变化对压力改变越敏感。γ_{D1}越大,井底压力越大,压力导数的数值也越大,如图 4 所示。

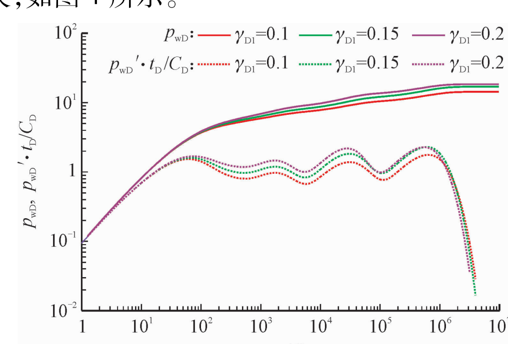

图 4 γ_{D1}对无因次井底压力及压力导数的影响

Fig. 4 The effect of γ_{D1} on dimensionless wellbore pressure and pressure-derivative

3)窜流系数 λ_{13}影响各窜流阶段发生的时间,其值越大,溶洞向裂缝窜流阶段进行的时间越短,基质向裂缝窜流阶段进行的时间越长而且效果越明显,

如图 5 所示。

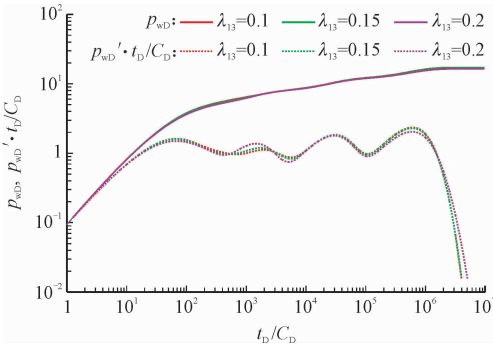

图 5　λ_{13}对无因次井底压力及压力导数的影响

Fig. 5　The effect of λ_{13} on dimensionless wellbore pressure and pressure-derivative

4)储容比 ω_i影响各窜流阶段出现的早晚和持续时间,图 6 给出了 ω_i对曲线形态的影响,ω_2不变,ω_1越小 ω_3越大,溶洞向裂缝窜流阶段持续时间越长;基质向裂缝窜流阶段出现得越晚,持续时间越短。

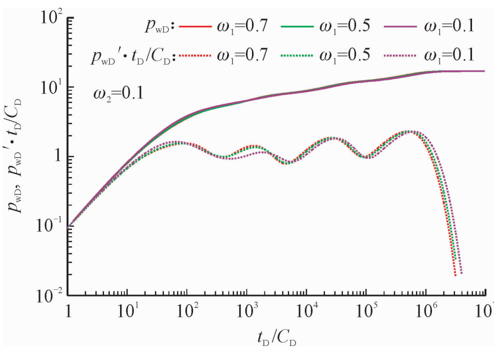

图 6　ω_i对无因次井底压力及压力导数的影响

Fig. 6　The effect of ω_i on dimensionless wellbore pressure and pressure-derivative

5)井筒储集系数 C_D对早期曲线的形态影响比较大,C_D越大,早期的井底压力越小,井筒储存阶段持续时间越长;溶洞向裂缝窜流阶段出现得越晚,持续时间越短而且窜流效果越弱;基质向裂缝窜流阶段出现得越晚,持续时间越短,如图 7 所示。

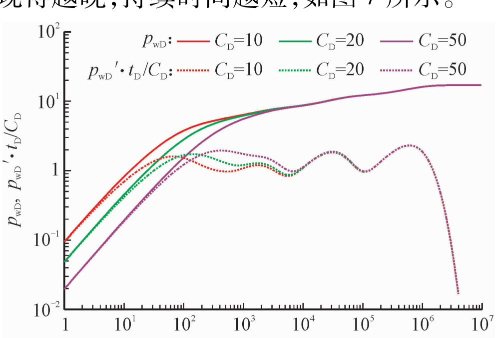

图 7　C_D对无因次井底压力及压力导数的影响

Fig. 7　The effect of C_D on dimensionless wellbore pressure and pressure-derivative

6)表皮系数 S 越大,井底压力越大;表皮因子影响阶段持续的时间越长而且压力导数值越大;溶洞向裂缝窜流阶段出现得越晚,持续时间越短而且效果越弱,如图 8 所示。

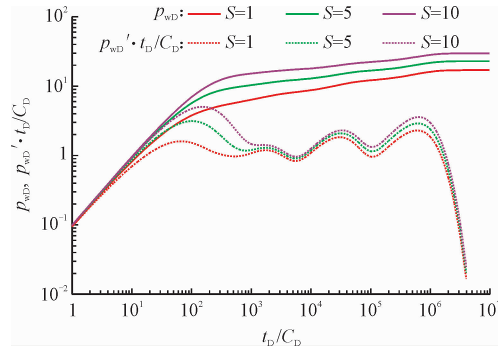

图 8　S 对无因次井底压力及压力导数的影响

Fig. 8　The effect of S on dimensionless wellbore pressure and pressure-derivative

4　结　论

1)碳酸盐岩油藏考虑为三渗三重介质建立试井模型,压力响应分为 6 个阶段:井筒储存阶段,表皮因子影响阶段,溶洞向裂缝窜流阶段,基质向裂缝窜流阶段,基质向溶洞窜流阶段和稳定流动阶段。

2)各个参数对压力响应曲线的各阶段有不同的影响。注蒸汽对储层进行改造后,注蒸汽后流体的流动性增强,定量生产时消耗的地层能量低;井底压力升高;各窜流阶段出现得越早,而且会缩短溶洞向裂缝窜流进行的时间并减弱其效果;压力导数提前衰减。

3)不同储存中的稠油黏度对温度的敏感性不同,这种敏感性用 β_D来刻画,其绝对值越大,则黏度对温度的变化越敏感,β_D对井底压力及压力导数影响就越明显,差分离散后计算时迭代的次数也会增多。

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